



# Modeling and non Linear Dynamic Analysis of the Chaotic Colpitts Oscillator up to 1 GHz

C. Rouifed<sup>a,b</sup>, A. Ouslimani<sup>b</sup>, M. Laghrouche<sup>a</sup>

<sup>a</sup>Mouloud Mammeri University of Tizi Ouzou, Tizi Ouzou, Algeria.

<sup>b</sup>Ecole nationale supérieure de l'électronique et ses applications, Cergy, France

\*Corresponding author. [a.alipacha@gmail.com](mailto:a.alipacha@gmail.com)

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**Abstract.** In this paper, we consider a Colpitts oscillator as a model for nonlinear dynamic analysis. In particular, we perform a bifurcation analysis using a real and theoretical model of the Colpitts oscillator. This analysis, simulated with Matlab, shows a difference between the two models while calculating their parameters. Moreover, in order to fix the optimal values of the circuit's component, spectrum simulation under ADS have been performed up to 1GHz. It shows a chaos bandwidth of 600 MHz.

**Keywords.** Colpitts oscillator, Bifurcation diagram, Chaotic system.

## INTRODUCTION

The first demonstration of a chaotic system generated by a linear circuit has been achieved in 1983, it is the so-called "Chua's circuits" (Chua, 1994). On the other hand, this chaotic signal could be also generated using non-linear systems, however oscillators are strongly depending on the initial conditions of the system but could be controlled to achieve the desired chaotic signal (Chua, 1994; Kennedy, 1994). This important propriety paves the way to introduce such signals in the information coding system. Among the electronic circuits that fulfill this function, Colpitts oscillator is an interesting circuit with high non-linearity behavior and wide bandwidth (Chen et al., 2014 ; Liao et al., 2016 ; Tamasevicius et al., 2001)..

These proprieties are the key enabling factors to implement such circuits in communication systems addressing coding and modulation (Volkovskii et al., 2005). It has been demonstrated in the bifurcation theory, the normal forms and the technique of communication could be useful to qualitatively characterize the different dynamic behavior shown by this oscillator (De Feo and Maggio, 2003). The approach that has been followed to obtain different equilibrium behaviors consists in selecting a model of the circuit which minimizes the main characteristics of the real Colpitts oscillator. Furthermore, we emphasize that the complex bifurcation structure exposed by the Colpitts oscillator gives a link to coexistence phenomena in a large area of parameter space. This paper is organized as flow: section 2: the real model of Colpitts oscillator and the bifurcation diagram are studied. Section 3: we present the ideal

model and we give the model based in ADS. As a conclusion, a comparison between the two models is reported in section 4.

### NONIDEAL MODEL

We consider the circuit configuration depicted in Figure. 1. The circuit is biased by  $V_{cc}$  and the current  $I_0$  with a conductance  $G_0$  (De Feo and Maggio, 2003).

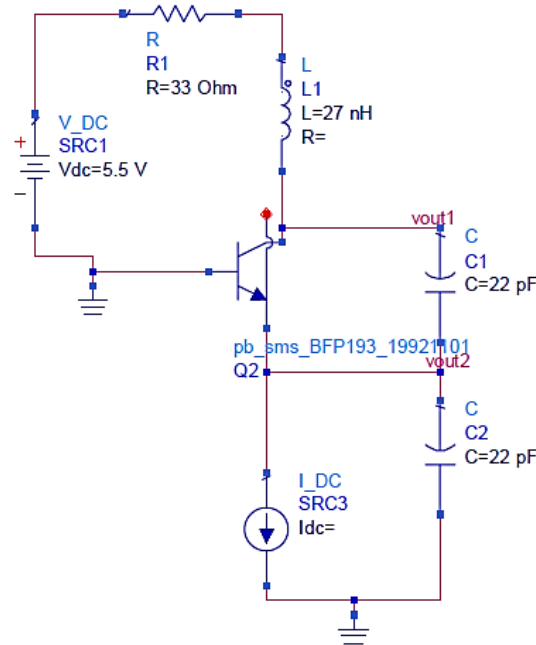


Fig. 1. Circuit diagram of chaotic Colpitts oscillator.

We consider the passive elements and the active element are neglected and the nonlinear model of the emitter base is presented by the following equation (Chua, 1994) :

$$f(-V_{c2}) = I_s \exp\left(\frac{-V_{c2}}{V_T}\right) \quad (1)$$

With  $\alpha$  is common base forward short-circuit current gain of the transistor. After some transformations in the equations of states for the schematic in figure 1 (Feo et al., 2000), they can be written as follows:

$$\begin{cases} \dot{x} = \frac{g^*}{Q(1-k)} [-\alpha * \eta(y) + z] \\ \dot{y} = \frac{g^*}{Qk} [(1-\alpha) * \eta(y) + z] - Q_0(1-k)y \\ \dot{z} = -\frac{Qk(1-k)}{g^*} [x + y] - \frac{1}{Q}Z \end{cases} \quad (2)$$

$$\begin{cases} \eta(y) = \exp(-y) - 1 \\ k = \frac{C_2}{C_1 + C_2} \\ Q_0 = G_0 W_0 \\ Q = \frac{W_0 L}{R} \\ g^* = \frac{L I_0}{(C_1 + C_2) R_1 V_T} \end{cases} \quad (3)$$

### Bifurcation diagram for reel circuit

In this part, we will analyze the bifurcation diagram of a real model and compare the results with the ideal model.

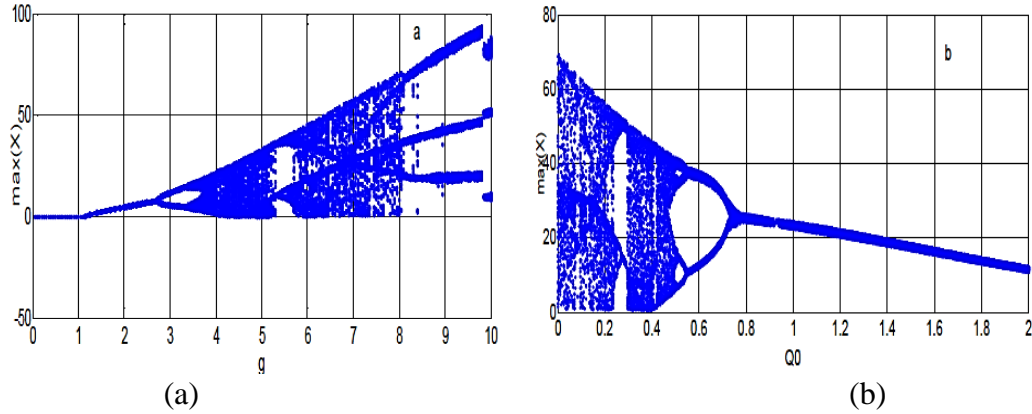


Fig.2. Bifurcation diagram real circuit : (a). As a function of the parameter  $g^*$ , (b) as a function of the parameter  $Q_0$ .

This diagram is obtained by plotting the maxima of the state  $x_2$  as a function of the two parameters  $g$  and  $q_0$  and all the other parameters are fixed. For the values of  $g^*=1.2$ ;  $Q_0=0.8$ , the condition of Barkhausen is satisfied and it presents the first sinusoidal oscillation. The first doubling of period will appear for  $g^*=2.7$ ;  $Q_0=0.75$ . This bifurcation continues to a critical value of  $g^*=3.6$ ,  $Q_0=0.45$  corresponding to the appearance of a chaotic behavior.

### IDEAL MODEL

For  $G_0 \rightarrow 0 \Rightarrow Q_0 \rightarrow 0$ , and  $\alpha = 1$ , the real model will be idealized and the number of parameters will be reduced, consequently, the analysis of the system will be simplified. In this case, the system depends only on  $g$ , this parameter allows a physical interpretation in terms of ideal model of oscillator, and mainly  $g$  defines the oscillation conditions and satisfies Criterion of Barkhausen (Feo et al., 2000). We note that idealizing circuit does not affect the dynamics of the oscillator.

$$\begin{cases} \dot{x} = \frac{g^*}{Q(1-k)} [-\eta(y) + z] \\ \dot{y} = \frac{g^*}{Qk} z \\ \dot{z} = -\frac{Qk(1-k)}{g^*} [x + y] - \frac{1}{Q} z \end{cases} \quad (4)$$

### Bifurcation diagram for ideal circuit

Figure 3 shows the bifurcation diagram of the ideal circuit with  $g^*$  parameter dependence. For  $g^*=1.1$  a sinusoidal oscillation is obtained corresponding to a limit cycle. When the value of  $g$  increases gradually, the changes have occurred at  $g^* = 2.6$ ,  $g^* = 3.5$  respectively. It is the transition from a two period oscillation to a chaotic oscillation through period doublings.

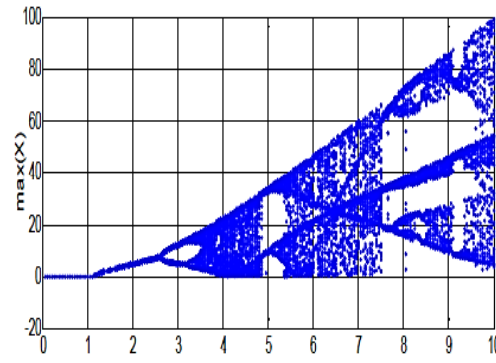


Fig.3. Bifurcation diagram for ideal model.

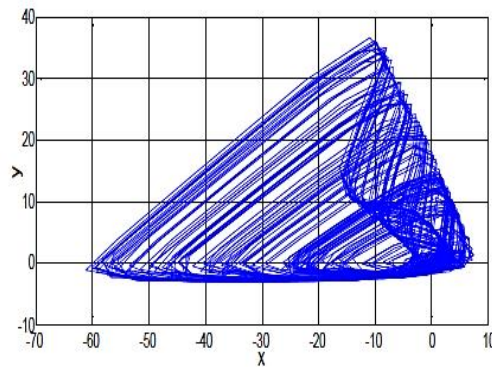


Fig. 4. The phase space of Colpitts oscillator.

Figure 4 illustrates the phase space of ideal model for value of  $g^* = 4.15$ , the strange attractor is obtained by plotting state  $y$  versus  $x$  of the system.

### Simulation results

To improve control of current  $I_0$  we replace the current source with voltage source  $V_2$  and resistance  $R_2$  as shown in figure 5. The transistor used, is a bipolar transistor (BFP193) with  $f_T$  of 8GHz. The transistor is described in the simulations using a nonlinear ADS model.

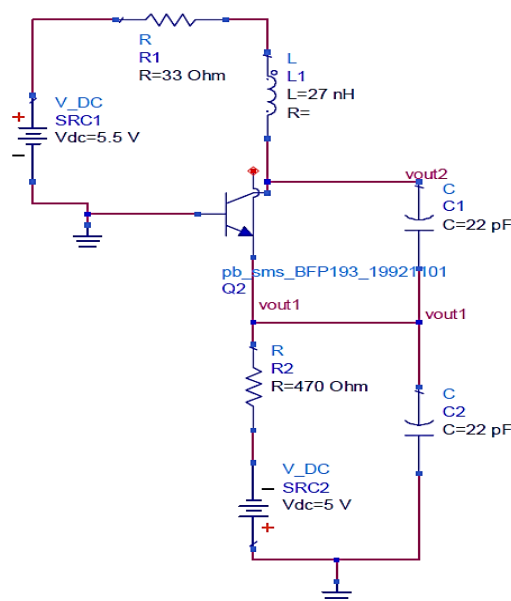


Fig.5. Simulation circuit of Colpitts oscillator.

The results of the simulations based on ADS model are shown by figure 6.

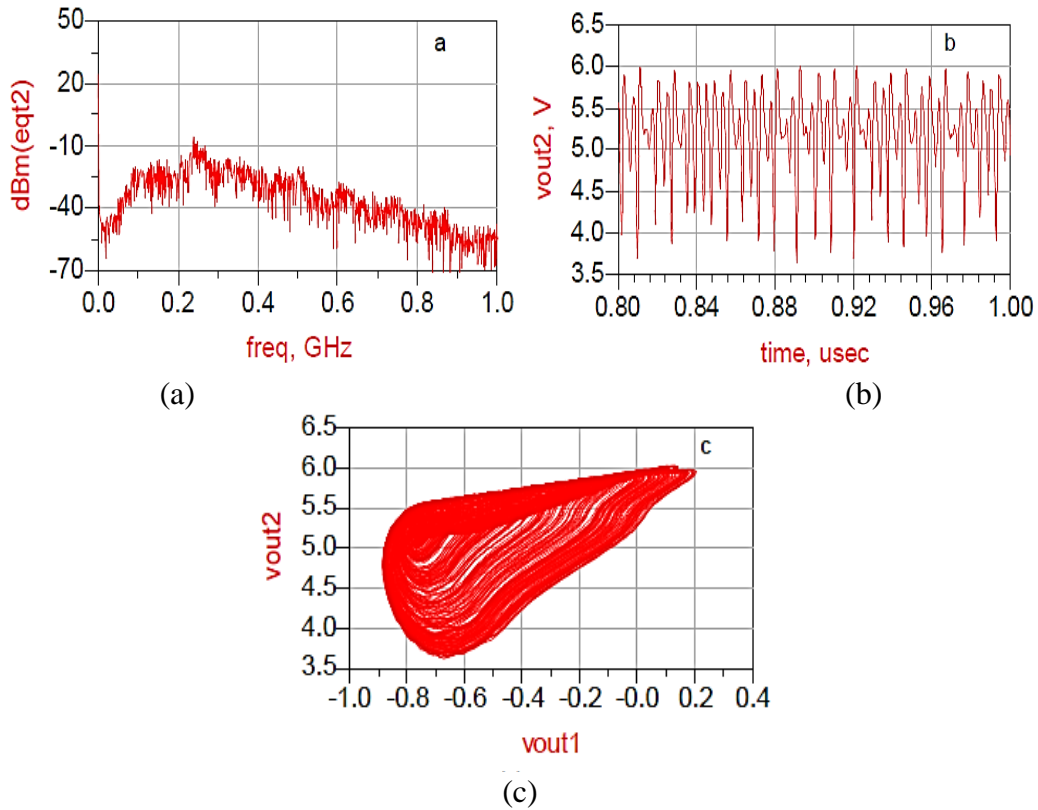


Fig.6. Simulation results: (a) frequency spectrum (b) output voltage (c) the phase diagram of Colpitts oscillator.

The elements of this circuit determines the fundamental frequency of the chaotic signal given by:

$$f_0 = \frac{1}{2\pi \sqrt{\frac{L_1 C_1 C_2}{C_1 + C_2}}} = 292 \text{ MHz} \quad (5)$$

The frequency spectrum is reported in figure 6 (a), it shows a spectrum up to 1 GHz with a large bandwidth of 600 MHz corresponding to different amplitudes (-50db to -10db). Figure 6 (b) shows the output signal of several  $V_{c2}$  as a function of the time and figure 6 (c) shows the phase diagram of the oscillator: the output  $V_{c2}$  corresponds to the emitter base voltage of the transistor ( $V_{c2} = -V_{be}$ ) and  $V_{c1}$  corresponds to the collector emitter voltage ( $V_{c1} = V_{ce}$ ).

## CONCLUSION

In this paper, we presented an analysis and comparison of the bifurcation diagram for Colpitts oscillator in both ideal and real circuits. This analysis reveals that the main difference between the two models remains on how the harmonic cycle is created. In other words, we can say that in the real model, the harmonic cycle incurs bifurcations similar to of the one in the ideal model with a slight difference in the value of the parameter  $g^*$ .

Therefore, the analysis results of the simplified ideal Colpitts oscillator are also qualitatively valid for the actual model. On the other hand, the validity of our model is limited by the operating frequency, because when this later increases, the model should take into account another parameters of the transistor (such as the capacitance  $C_{be}$ ,  $C_{ce}$  which will be included in parallel with  $C_1$  and  $C_2$  respectively) in order to have more accurate results.

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